

SECURE PARAMETER GENERATING DEVICE AND PARAMETER  
GENERATING METHOD IN ALGEBRAIC CURVE CRYPTOGRAPHY

BACKGROUNDS OF THE INVENTION

5                   FIELD OF THE INVENTION

The present invention relates to a secure  
parameter generating device, a generating method, and a  
storing medium in a discrete logarithm cryptography  
(hereinafter, referred to an algebraic curve  
10 cryptography), and more particularly to a secure  
parameter generating device and its generating method in  
a discrete logarithm cryptography using Jacobian group  
of algebraic curve.

DESCRIPTION OF THE RELATED ART

15                   A discrete logarithm cryptography is a public  
key system based on the difficulty of a discrete  
logarithm problem on a given finite field. In order to  
keep the security of cryptography, the order of the  
finite field must be almost a prime number, that is, a  
20 factor of small integer and large integer. The algebraic  
curve cryptography that is one of the discrete logarithm  
cryptography needs to use an algebraic curve such that  
the order of the Jacobian group is almost a prime number.

25                   In the case of an elliptic curve that is the  
simplest algebraic curve, an efficient algorithm of  
calculating the order of the Jacobian group over any  
elliptic curve is known. The detailed description is

shown in, for example, "Counting points on elliptic curves over finite fields", Journal de Theorie des Nombres, de Bordeaux 7 (1995), 219-254, Institut de Mathematique de Bordeaux, written by Rene Schoof. The  
5 elliptic curve such that the order of the Jacobian group is almost a prime factor can be obtained as follows, by using the above algorithm.

1. Generate a random elliptic curve E.
2. Calculate the order n of the Jacobian group  
10 of E.
3. If n is almost a prime number, output E; otherwise, return to 1.

In the case of an algebraic curve other than an  
15 elliptic curve, no efficient algorithm of calculating the order of the Jacobian group is known except for one hyper-elliptic curve. Therefore, the algebraic curve which can be used in the algebraic curve cryptography is limited to an elliptic curve or one exceptional hyper  
20 elliptic curve.

As for the h-fold operation of the elements in the Jacobian group, "Software Installation of Discrete Logarithm Cryptography Using  $C_{ab}$  curve" written by Arita, Yoshikawa, and Miyauchi, pp.573-578, Security Symposium  
25 on Cryptography and Information in 1999, is known.

Further, the technique disclosed in Japanese Patent Publication Laid-Open (Kokai) No. Heisei 6-282226

comprises a step of selecting any prime number, storing  
an encryption key corresponding to the prime number into  
the public file device, generating a decoding key list  
corresponding to the prime number and the encryption key,  
5 and storing the decoding key list together with the  
prime number into a decoder, wherein an encoder obtains  
a public key of a receiver (decoder) from the public  
file, to multiply the plaintext on an elliptic curve,  
its value is sent to the decoder as a cryptogram, and  
10 the decoder computes a parameter of the elliptic curve  
from the cryptogram and selects a decoding key  
corresponding to the parameter by use of the decoding  
key list, thereby obtaining the plaintext from the value  
obtained by multiplying the cryptogram by the elliptic  
15 curve, using Chinese residue theorem.

The above mentioned conventional technique  
limits the usable algebraic curves to an elliptic curve  
or one of exceptional hyper-elliptic curve. Since the  
elliptic curve and the hyper-elliptic curve are  
20 extremely particular algebraic curve from the viewpoint  
of the whole algebraic curves and this narrows the  
target for cryptanalysis, there arises a security  
problem of an algebraic curve cryptography.

#### SUMMARY OF THE INVENTION

An object of the present invention is to make it  
possible to use a higher and complicated algebraic curve

which couldn't be used, for an algebraic curve cryptography, and to provide a secure parameter generating device in an algebraic curve cryptography for improving the security of the algebraic curve cryptography.

Further, another object of the present invention is to make it possible to use a higher and complicated algebraic curve which couldn't be used, for an algebraic curve cryptography, and to provide a secure parameter generating method in the algebraic cryptography for improving the security of the algebraic curve cryptography.

According to the first aspect of the invention, a secure parameter generating device in an algebraic curve cryptography, comprises

an input means for receiving two different prime numbers (a, b) specifying degree of complexity of a curve and size (n) of an encryption key to be used,

a Stickelberger element computing device for computing a Stickelberger element ( $\omega$ ) in an ab cyclotomic, based on the prime number (a) and the prime number (b),

a Jacobian addition candidate value computing device for computing Jacobian addition candidate value j corresponding to the two different prime numbers a and b, and a prime number p corresponding to the Jacobian addition candidate value j, based on the prime number

(a), the prime number (b), the size (n) of an encryption key, and the Stickelberger element ( $\omega$ ) ,

an order candidate value computing device for computing a class H consisting of a plurality of

5 candidate values for order of a Jacobian group of an algebraic curve specified by the prime number a and the prime number b, based on the prime number a, the prime number b, and the Jacobian addition candidate value j,

10 a security judging device for searching for a candidate value h meeting a security condition such as almost prime number characteristic from the class H, according to the class H,

15 a parameter deciding device for computing a parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value h, of the algebraic curves specified by the prime number a, the prime number b, and the prime number p, based on the prime number a, the prime number b, the prime number p, and the candidate value h, and

20 an output device for supplying the parameter of the algebraic curve computed by said parameter deciding device.

25 In the preferred construction, a secure parameter generating device in an algebraic curve cryptography further comprises

an a-storing means, a b-storing means, and an n-storing means for respectively storing the prime number

a, the prime number b, and the size n of the encryption key received by said input means,

a  $\omega$ -storing means for storing a Stickelberger element  $\omega$  computed by said Stickelberger element computing device,

a p-storing means and a j-storing means for respectively storing the prime number p and the Jacobian addition candidate value j computed by said Jacobian addition candidate value computing device,

an H-storing means for storing the class H computed by said order candidate value computing device, and

an h-storing means for storing the candidate value h found by said security judging device.

In another preferred construction, a secure parameter generating device in an algebraic curve cryptography comprises

said Stickelberger element computing device for computing the Stickelberger element  $\omega$  by use of the equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}}$  (where, t runs on a typical series of irreducible residue class with ab used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$  indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the ab cyclotomic ( $\zeta$  is the primitive ab root of 1)), based on the prime number a and the prime number b.

In another preferred construction, a secure parameter generating device in an algebraic curve cryptography comprises

5 said Jacobian addition candidate value computing device for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the  
10 prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ .

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20 based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ .

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In another preferred construction, a secure parameter generating device in an algebraic curve cryptography comprises

said parameter deciding device for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on the prime number  $a$ , the prime number  $b$ , the prime number

p, and the candidate value h, generating a random point G over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer l from 1 to a inclusively and each integer m from 1 to b inclusively, computing the h-fold of an element in the Jacobian group indicated by the point G, and supplying p,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value h, of the algebraic curves specified by the prime number a and the prime number b if the result is equal to an identity element in the Jacobian group.

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the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$  inclusively and each integer  $m$  from 1 to  $b$  inclusively, computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

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used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$  indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the ab cyclotomic ( $\zeta$  is the primitive ab root of 1)), based on the prime number  $a$  and the prime number  $b$ ,

said Jacobian addition candidate value computing device for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive ab root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ ,

said order candidate value computing device for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{\{K|Q\}}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive ab root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ , and

said parameter deciding device for requiring the

primitive a root  $\zeta_a$  and the primitive b root  $\zeta_b$  of 1 with the prime number p used as the divisor, based on the prime number a, the prime number b, the prime number p, and the candidate value h, generating a random point G over an algebraic curve defined by the equation  $\zeta_a^{-1} Y^a + \zeta_b^m X^b + 1 = 0$ , as for each integer l from 1 to a inclusively and each integer m from 1 to b inclusively, computing the h-fold of an element in the Jacobian group indicated by the point G, and supplying p,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value h, of the algebraic curves specified by the prime number a and the prime number b if the result is equal to an identity element in the Jacobian group.

According to the second aspect of the invention, a secure parameter generating method in an algebraic curve, comprises the steps of

a Stickelberger element computing procedure for computing a Stickelberger element  $\omega$  in an ab cyclotomic, respectively based on two different prime numbers a and b specifying degree of complexity of curve,

a Jacobian addition candidate value computing procedure for computing Jacobian addition candidate value j corresponding to the two different prime numbers a and b, and a prime number p corresponding to the Jacobian addition candidate value j, respectively based on the prime number a, the prime number b, the size n of

an encryption key, and the Stickelberger element  $\omega$ ,

an order candidate value computing procedure for computing a class  $H$  consisting of a plurality of candidate values for order of a Jacobian group of an algebraic curve specified by the prime number  $a$  and the prime number  $b$ , respectively based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ ,

a security judging procedure for searching for a candidate value  $h$  meeting a security condition such as almost prime number characteristic from the class  $H$ , according to the class  $H$ , and

a parameter deciding procedure for computing a parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$ , the prime number  $b$ , and the prime number  $p$ , respectively based on the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ .

In the preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

a procedure for storing a Stickelberger element  $\omega$  computed by said Stickelberger element computing procedure into said  $\omega$ -storing means,

a procedure for respectively storing the prime number  $p$  and the Jacobian addition candidate value  $j$

computed by said Jacobian addition candidate value  
computing procedure into said p-storing means and j-  
storing means,

5 a procedure for storing the class H computed by  
said order candidate value computing procedure into said  
H-storing means, and

a procedure for storing the candidate value h  
found by said security judging procedure into said h-  
storing means.

10 In another preferred construction, a secure  
parameter generating method in an algebraic curve  
cryptography comprises

15 said Stickelberger element computing procedure  
for computing the Stickelberger element  $\omega$  by use of the  
equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}}$  (where, t runs  
on a typical series of irreducible residue class with ab  
used as a divisor,  $[\lambda]$  indicates the maximum integer not  
exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a  
fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$   
20 indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the ab cyclotomic  
( $\zeta$  is the primitive ab root of 1)), based on the prime  
number a and the prime number b.

25 In another preferred construction, a secure  
parameter generating method in an algebraic curve  
cryptography comprises

said Jacobian addition candidate value computing  
procedure for generating  $\alpha$  at random, which is an

algebraic integer  $\gamma$  generating a prime ideal of a  
cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and  
whose absolute norm becomes the prime number  $p$  of bit  
length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ ,  
5 the prime number  $b$ , the size  $n$  of the encryption key,  
and the Stickelberger element  $\omega$ , and computing the  
Jacobian addition candidate value  $j$  by use of the  
equation  $j = \gamma^\omega$ .

In another preferred construction, a secure  
10 parameter generating method in an algebraic curve  
cryptography comprises

said Stickelberger element computing procedure  
for computing the Stickelberger element  $\omega$  by use of the  
equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}}$  (where,  $t$  runs  
15 on a typical series of irreducible residue class with  $ab$   
used as a divisor,  $[\lambda]$  indicates the maximum integer not  
exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a  
fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$   
indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the  $ab$  cyclotomic  
20 ( $\zeta$  is the primitive  $ab$  root of 1)), based on the prime  
number  $a$  and the prime number  $b$ , and

said Jacobian addition candidate value computing  
procedure for generating  $\alpha$  at random, which is an  
algebraic integer  $\gamma$  generating a prime ideal of a  
25 cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and  
whose absolute norm becomes the prime number  $p$  of bit  
length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ ,



the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ .

5                   In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

said order candidate value computing procedure for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ .

15                   In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

20                   said Stickelberger element computing procedure for computing the Stickelberger element  $\omega$  by use of the equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}}$  (where,  $t$  runs on a typical series of irreducible residue class with  $ab$  used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$

indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the ab cyclotomic ( $\zeta$  is the primitive ab root of 1)), based on the prime number  $a$  and the prime number  $b$ , and

said order candidate value computing procedure  
5 for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1  
10 to  $2ab$  inclusively, when  $\zeta$  is the primitive ab root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ .

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procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a  
20 cyclotomic  $K$  generated by the primitive ab root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the  
25 Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ , and

said order candidate value computing procedure

for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ .

In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

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said Jacobian addition candidate value computing procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit

length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ ,  
the prime number  $b$ , the size  $n$  of the encryption key,  
and the Stickelberger element  $\omega$ , and computing the  
Jacobian addition candidate value  $j$  by use of the  
equation  $j = \gamma^\omega$ , and

said order candidate value computing procedure  
for computing a candidate value  $h_k$  for the order of the  
Jacobian group of an algebraic curve specified by the  
parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab  
cyclotomic  $K$ ), as for each  $k$  that is an integer from 1  
to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1,  
based on the prime number  $a$ , the prime number  $b$ , and the  
Jacobian addition candidate value  $j$ , and computing the  
class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ .

In another preferred construction, a secure  
parameter generating method in an algebraic curve  
cryptography comprises

said parameter deciding procedure for requiring  
the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1  
with the prime number  $p$  used as the divisor, based on  
the prime number  $a$ , the prime number  $b$ , the prime number  
 $p$ , and the candidate value  $h$ , generating a random point  
 $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a$   
 $+ \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$   
inclusively and each integer  $m$  from 1 to  $b$  inclusively,  
computing the  $h$ -fold of an element in the Jacobian group

indicated by the point G, and supplying  $p$ ,  $\zeta_a^1$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

said Stickelberger element computing procedure for computing the Stickelberger element  $\omega$  by use of the equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}} \{ -t^{-1} \}$  (where,  $t$  runs on a typical series of irreducible residue class with  $ab$  used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$  indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the  $ab$  cyclotomic ( $\zeta$  is the primitive  $ab$  root of 1)), based on the prime number  $a$  and the prime number  $b$ , and

said parameter deciding procedure for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^1 y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$  inclusively and each integer  $m$  from 1 to  $b$  inclusively,

computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^1$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

said Jacobian addition candidate value computing procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ , and

said parameter deciding procedure for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^1 y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$

inclusively and each integer  $m$  from 1 to  $b$  inclusively, computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

said order candidate value computing procedure for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ , and

said parameter deciding procedure for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a$

+  $\zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$  inclusively and each integer  $m$  from 1 to  $b$  inclusively, computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^l$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

said Stickelberger element computing procedure for computing the Stickelberger element  $\omega$  by use of the equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t^{-1}}$  (where,  $t$  runs on a typical series of irreducible residue class with  $ab$  used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$  indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the  $ab$  cyclotomic ( $\zeta$  is the primitive  $ab$  root of 1)), based on the prime number  $a$  and the prime number  $b$ ,

said Jacobian addition candidate value computing procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit



length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ ,  
the prime number  $b$ , the size  $n$  of the encryption key,  
and the Stickelberger element  $\omega$ , and computing the  
Jacobian addition candidate value  $j$  by use of the  
equation  $j = \gamma^\omega$ , and

said parameter deciding procedure for requiring  
the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1  
with the prime number  $p$  used as the divisor, based on  
the prime number  $a$ , the prime number  $b$ , the prime number  
 $p$ , and the candidate value  $h$ , generating a random point  
 $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a$   
 $+ \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$   
inclusively and each integer  $m$  from 1 to  $b$  inclusively,  
computing the  $h$ -fold of an element in the Jacobian group  
indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^{-1}$ , and  $\zeta_b^m$   
as the parameter of an algebraic curve whose order of  
the Jacobian group is in accord with the candidate value  
 $h$ , of the algebraic curves specified by the prime number  
 $a$  and the prime number  $b$  if the result is equal to an  
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equation  $\omega = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{\{-t^{-1}\}}$  (where,  $t$  runs  
on a typical series of irreducible residue class with  $ab$

used as a divisor,  $[\lambda]$  indicates the maximum integer not exceeding a rational number  $\lambda$ ,  $\langle \lambda \rangle$  indicates a fractional portion  $\lambda - [\lambda]$  of the rational number  $\lambda$ ,  $\sigma_t$  indicates Galois mapping  $\zeta \rightarrow \zeta^t$  in the ab cyclotomic  
5 ( $\zeta$  is the primitive ab root of 1)), based on the prime number  $a$  and the prime number  $b$ ,

said order candidate value computing procedure for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the  
10 parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{\{K|Q\}}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive ab root of 1, based on the prime number  $a$ , the prime number  $b$ , and the  
15 Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ , and

said parameter deciding procedure for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on  
20 the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$  inclusively and each integer  $m$  from 1 to  $b$  inclusively,  
25 computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of

the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

5            In another preferred construction, a secure parameter generating method in an algebraic curve cryptography comprises

             said Jacobian addition candidate value computing procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ ,

             said order candidate value computing procedure for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the ab cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1, based on the prime number  $a$ , the prime number  $b$ , and the Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ , and

said parameter deciding procedure for requiring the primitive a root  $\zeta_a$  and the primitive b root  $\zeta_b$  of 1 with the prime number p used as the divisor, based on the prime number a, the prime number b, the prime number p, and the candidate value h, generating a random point G over an algebraic curve defined by the equation  $\zeta_a^{-1} y^a + \zeta_b^m x^b + 1 = 0$ , as for each integer l from 1 to a inclusively and each integer m from 1 to b inclusively, computing the h-fold of an element in the Jacobian group indicated by the point G, and supplying p,  $\zeta_a^{-1}$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value h, of the algebraic curves specified by the prime number a and the prime number b if the result is equal to an identity element in the Jacobian group.

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number  $a$  and the prime number  $b$ ,

said Jacobian addition candidate value computing procedure for generating  $\alpha$  at random, which is an algebraic integer  $\gamma$  generating a prime ideal of a

5 cyclotomic  $K$  generated by the primitive  $ab$  root of 1 and whose absolute norm becomes the prime number  $p$  of bit length  $2n/(a-1)(b-1)$  or so, based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of the encryption key, and the Stickelberger element  $\omega$ , and computing the  
10 Jacobian addition candidate value  $j$  by use of the equation  $j = \gamma^\omega$ ,

said order candidate value computing procedure for computing a candidate value  $h_k$  for the order of the Jacobian group of an algebraic curve specified by the  
15 parameters  $a$  and  $b$ , using the equation  $h_k = \text{Norm}_{K|Q}(1 + (-\zeta)^k j)$  (where  $\text{Norm}_{K|Q}$  is a norm mapping in the  $ab$  cyclotomic  $K$ ), as for each  $k$  that is an integer from 1 to  $2ab$  inclusively, when  $\zeta$  is the primitive  $ab$  root of 1, based on the prime number  $a$ , the prime number  $b$ , and the  
20 Jacobian addition candidate value  $j$ , and computing the class of the candidate values,  $H = \{h_1, h_2, \dots, h_{2ab}\}$ , and

said parameter deciding procedure for requiring the primitive  $a$  root  $\zeta_a$  and the primitive  $b$  root  $\zeta_b$  of 1 with the prime number  $p$  used as the divisor, based on  
25 the prime number  $a$ , the prime number  $b$ , the prime number  $p$ , and the candidate value  $h$ , generating a random point  $G$  over an algebraic curve defined by the equation  $\zeta_a^{-1} Y^a$

+  $\zeta_b^m x^b + 1 = 0$ , as for each integer  $l$  from 1 to  $a$  inclusively and each integer  $m$  from 1 to  $b$  inclusively, computing the  $h$ -fold of an element in the Jacobian group indicated by the point  $G$ , and supplying  $p$ ,  $\zeta_a^l$ , and  $\zeta_b^m$  as the parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$  and the prime number  $b$  if the result is equal to an identity element in the Jacobian group.

According to another aspect of the invention, a computer readable memory storing a program for generating a secure parameter in an algebraic curve cryptography, to run the program on a computer,

the program comprises the steps of

a Stickelberger element computing procedure for computing a Stickelberger element  $\omega$  in an ab cyclotomic, respectively based on two different prime numbers  $a$  and  $b$  specifying degree of complexity of curve,

a Jacobian addition candidate value computing procedure for computing Jacobian addition candidate value  $j$  corresponding to the two different prime numbers  $a$  and  $b$ , and a prime number  $p$  corresponding to the Jacobian addition candidate value  $j$ , respectively based on the prime number  $a$ , the prime number  $b$ , the size  $n$  of an encryption key, and the Stickelberger element  $\omega$ ,

an order candidate value computing procedure for computing a class  $H$  consisting of a plurality of

candidate values for order of a Jacobian group of an algebraic curve specified by the prime number a and the prime number b, respectively based on the prime number a, the prime number b, and the Jacobian addition candidate value j,

a security judging procedure for searching for a candidate value h meeting a security condition such as almost prime number characteristic from the class H, according to the class H, and

a parameter deciding procedure for computing a parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value h, of the algebraic curves specified by the prime number a, the prime number b, and the prime number p, respectively based on the prime number a, the prime number b, the prime number p, and the candidate value h.

Other objects, features and advantages of the present invention will become clear from the detailed description given herebelow.

#### BRIEF DESCRIPTION OF THE DRAWINGS

The present invention will be understood more fully from the detailed description given herebelow and from the accompanying drawings of the preferred embodiment of the invention, which, however, should not be taken to be limitative to the invention, but are for explanation and understanding only.

In the drawings:

Fig. 1 is a block diagram showing the form of the first embodiment according to the present invention;

Fig. 2 is a flow chart showing the operation of a Stickelberger element computing device;

Fig. 3 is a flow chart showing an operation of the Jacobian additive candidate value computing device;

Fig. 4 is a flow chart showing the operation of the order candidate value computing device;

Fig. 5 is a flow chart showing the parameter deciding device;

Fig. 6 is a block diagram showing the form of the third embodiment of the present invention.

#### DESCRIPTION OF THE PREFERRED EMBODIMENT

The preferred embodiment of the present invention will be discussed hereinafter in detail with reference to the accompanying drawings. In the following description, numerous specific details are set forth in order to provide a thorough understanding of the present invention. It will be obvious, however, to those skilled in the art that the present invention may be practiced without these specific details. In other instance, well-known structures are not shown in detail in order to unnecessary obscure the present invention.

First, the principle of the present invention will be described.



The present invention is to efficiently search for an algebraic curve such that the order of the Jacobian group is almost a prime number, from the class of an algebraic curve having the definition expression such as  $\alpha Y^a + \beta X^b + 1 = 0$ , and to make it possible to use a higher and complicated algebraic curve that couldn't be used, for an algebraic curve cryptography. Here, the parameters  $a$  and  $b$  indicate the degree of complexity of curves.

The algebraic curve over a finite field  $F_q$  of the order  $q$ , which has the definition expression such as  $\alpha Y^a + \beta X^b + 1 = 0$ , is defined as  $C(q, \alpha, \beta)$ . As for the algebraic curve  $C(q, \alpha, \beta)$ , the order of the Jacobian group can be designed using the description about its  $L$  function by use of the Jacobian addition.

Hereafter, for brief description, assume that  $q := p$  ( $p$  is expressed by  $q$ ) is a prime number, and that the expression,  $p \equiv 1 \pmod{\text{LCM}(a, b)}$  is satisfied ( $\text{LCM}$  is the least common multiple). Further, the primitive  $ab$  root of 1 is defined as  $\zeta$ . The prime number  $p$  is completely factorized into  $m$  pieces of prime ideals,  $P_1, P_2, \dots, P_m$  in a cyclotomic  $Q(\zeta)$ . Here, the number  $m$  is the number of irreducible residue classes of the divisor  $ab$ .

The generator  $w$  of the multiplication group  $F_p^*$  of the finite field  $F_p$  is fixed, and the index  $\chi_s$  of  $F_p^*$ , as for the rational number such that  $(p-1)s$  becomes an

integer, is defined as  $\chi_s(w) = \exp(2\pi i s)$  (where  $i$  is an imaginary number). Assuming that  $\chi_s(0) = 0$  ( $s$ : when it is not an integer),  $=1$  ( $s$ : when it is an integer), the domain is expanded on the whole  $F_p$ . As for the integer  $l=1, 2, \dots, a-1$  and the integer  $m=1, 2, \dots, b-1$ , the expression,  $j_p(l, m) = \sum_{\{l+v_1+v_2=0\}} \chi_{l/a}(v_1) \chi_{m/b}(v_2)$  is called as Jacobian addition. Where,  $v_1$  and  $v_2$  run on  $v_1, v_2 \in F_p$  meeting the equation  $l+v_1+v_2=0$ . At this time, it is well known that the  $L$  function  $L_p(U)$  of  $C(p, \alpha, \beta)$  can be expressed by using the Jacobian addition as follows.

$$L_p(U) = \prod_{l=1, 2, \dots, a-1, m=1, 2, \dots, b-1} (1 + \chi_{l/a}(\alpha^{-1}) \chi_{m/b}(\beta^{-1}) j_p(l, m) U).$$

Therefore, the order  $h$  of the Jacobian group of  $C(p, \alpha, \beta)$  is given by  $h = L_p(1) = \prod_{l=1, 2, \dots, a-1, m=1, 2, \dots, b-1} (1 + \chi_{l/a}(\alpha^{-1}) \chi_{m/b}(\beta^{-1}) j_p(l, m))$ . It is necessary to calculate the Jacobian addition  $j_p(l, m)$ , in order to require the order of the Jacobian group.

However, since it is impossible to directly calculate the Jacobian addition  $j_p(l, m)$  according to the definition expression, from the viewpoint of the volume of calculation, the Stickelberger element as for the following Jacobian addition is used.

Assume that  $[\lambda]$  indicates the maximum integer not exceeding the rational number  $\lambda$  and that  $\langle \lambda \rangle$  indicates the fraction part  $\lambda - [\lambda]$  of the rational number  $\lambda$ . Further, assume that  $\sigma_t$  indicates the Galois mapping

$\zeta \rightarrow \zeta^t$  of the cyclotomic field  $Q(\zeta)$ . The Stickelberger element  $\omega(a, b)$  that is the generator of the group ring  $Z[\text{Gal}(Q(\zeta)|Q)]$  is defined as  $\omega(a, b) = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t}^{-1}$ . Where,  $t$  runs on the typical series of the irreducible residue class with the divisor  $ab$ .

It is well known that the expression  $(j_p(1, m)) = p^{\omega(a, b)}$  is satisfied as the ideal of the cyclotomic field  $Q(\zeta)$ . Where,  $P$  is the prime ideal on  $p$ . By the above expression,  $j_p(1, m)$  is necessarily resolved except for the  $2ab$  root of 1. Of the results, the degree of freedom for  $ab$  root can be obtained by the degree of freedom of the coefficient  $\alpha, \beta \in F_q$  of  $C(p, \alpha, \beta)$ .

In these ways, the search algorithm of the following secure curve  $C(p, \alpha, \beta)$  can be obtained. The search algorithm of the secure curve  $C(p, \alpha, \beta)$  input: the number of Jacobian bits  $n$

output:  $p, \alpha, \beta$

(1)  $g \leftarrow (a-1)(b-1)/2$

(2) The candidate  $j$  of the Jacobian addition as for the prime number  $p$  of some  $n/g$  bit or the like is searched for by using the calculation algorithm of the candidate value of the Jacobian addition as described later:

$(p, j) \leftarrow \{\text{calculation algorithm of the candidate value of the Jacobian addition}\}(n/g)$ .

(3) as for the respective  $k = 0, 1, \dots, ab$ ,

$h_k \leftarrow \prod_{l=1, 2, \dots, a-1, m=1, 2, \dots, b-1} (1 + (-\zeta)^k j)$

(4) Check whether there is an almost prime number  $h_k$  in

$\{ h_0, h_1, \dots, h_{ab} \}$ . If there is none, return to (1). If there is,  $h := h_k$  is defined.

(5) The symbols  $\zeta_a, \zeta_b$  are respectively defined as a-root of 1 and b-root of 1 in  $F_p$ . Check whether the order of the Jacobian group of the curve  $C(p, \zeta_a^{-1}, \zeta_b^m): \zeta_a^{-1} y^a + \zeta_b^m x^b + 1 = 0$  is equal to  $h$ , as for the respective  $l = 0, 1, \dots, a-1$  and the respective  $m = 0, 1, \dots, b-1$ . If it is equal, output  $p, \alpha = \zeta_a^{-1}, \beta = \zeta_b^m$  and finish the operation. If such  $l$  and  $m$  don't exist, return to (2).

10 In the calculation algorithm of the candidate value of the Jacobian addition used in the above, the candidate value of the Jacobian addition is required by using the above-mentioned Stickelberger element  $\omega(a, b) = \sum_t [\langle t/a \rangle + \langle t/b \rangle] \sigma_{-t}^{-1}$ .

15 The calculation algorithm of the candidate value of the Jacobian addition is as follows.

input: the number of bits  $m$ ,

output:  $p, j$ ,

(1)  $\omega \leftarrow \sum_t (\langle t/a \rangle + \langle t/b \rangle) \sigma_{-t}^{-1}$ ,

20 (2) Generate  $\gamma_0 = \sum_{i=0}^{m-1} c_i \zeta^i$  ( $-10 < c_i < 10$ ) at random.

(3) as for the respective  $i = 1, 2, \dots$ ,

$\gamma \leftarrow \gamma_0 + I$ ,

$p \leftarrow \text{Norm}_{\mathbb{Q}(\zeta)|\mathbb{Q}}(\gamma)$ ,

Is  $p$  smaller than  $m$  bit or so?

25 yes  $\rightarrow$  continue,

Is  $p$  larger than  $m$  bit or so?

yes  $\rightarrow$  to (2),

Is  $p$  a prime number?

no  $\rightarrow$  continue,

(4) Output  $j \leftarrow \gamma^{\omega}$ ,  $p$  and  $j$ , and finish.

This time, the form of the first embodiment of  
5 the present invention will be described with reference  
to the accompanying drawings. Fig. 1 is a block diagram  
showing the form of the first embodiment.

With reference to Fig. 1, the form of the first  
embodiment of the present invention comprises a  
10 Stickelberger element computing device 11, a Jacobian  
addition candidate value computing device 12, an order  
candidate value computing device 13, a security judging  
device 14, a parameter deciding device 15, a memory 16,  
an input device 17, an output device 18, and a central  
15 processing unit 19.

The memory 16 includes a a-storing file 161, a  
b-storing file 162, a  $\omega$ -storing file 163, a j-storing  
file 164, an H-storing file 165, an h-storing file 166,  
a p-storing file 167, and an n-storing file 168.

20 Hereinafter, assume that a known method is used  
for the norm arithmetic  $N_{Q(\zeta)|Q}$  and four fundamental  
arithmetic rules of algebraic number in the cyclotomic  
field  $Q(\zeta)$ , arithmetic of the function of the Galois  
group  $G(Q(\zeta)|Q)$  for the cyclotomic field  $Q(\zeta)$ , and  
25 addition and multiplication in the group ring  $Z[G(Q(\zeta)|Q)]$   
on the Galois group  $G(Q(\zeta)|Q)$  of the integer ring  $Z$   
coefficient.

The operation of the form of a first embodiment according to the present invention will be described this time.

Fig. 2 is a flow chart showing the operation of the Stickelberger element computing device 11. Fig. 3 is a flow chart showing the operation of the Jacobian addition candidate value computing device 12. Fig. 4 is a flow chart showing the operation of the order candidate value computing device 13. Fig. 5 is a flow chart showing the operation of the parameter deciding device 15.

The description will be made in the case where two different prime numbers,  $a=3$ ,  $b=7$ , specifying the degree of complexity of a curve, and the size  $n=160$  of an encryption key to be used are supplied from the input device 17. The supplied  $a$  and  $b$  are temporarily stored in the  $a$ -storing file 161 and the  $b$ -storing file 162 respectively, through the central processing unit 19. Further, a variable found in the following description is stored in the memory 16.

The Stickelberger element computing device 11 obtains  $a=3$ ,  $b=7$  from the  $a$ -storing file 161 and the  $b$ -storing file 162, according to the operation as shown in Fig. 2, and operates as follows.

A typical series  $\{1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20\}$  of the irreducible residue class with  $a \cdot b = 3 \times 7 = 21$  used for the variable  $L$  as the divisor is stored

in Step S21 of Fig. 2.

As for the respective integers  $t$  included in the variable  $L=\{1, 2, 4, 5, 8, 10, 11, 13, 16, 17, 19, 20\}$  in Step 22 of Fig. 2, for example, as for  $t=1$ , since  
5  $[<1/3>+<1/7>] = [1/3+1/7]=[10/21]=0$ , 0 is stored in the variable  $m$ ; since  $-1^{-1} \equiv -1 \equiv 20 \pmod{21}$ , 20 is stored in the variable  $s$ ; since  $0 \times \sigma_{20} = 0$ , 0 is stored in the variable  $\lambda_1$ .

As for the other  $t$ , since

10  $[<2/3>+<2/7>]=[2/3+2/7]=[20/21]=0$ , 0 is stored in the variable  $\lambda_2$ ; since  $[<4/3>+<4/7>]=[1/3+4/7]=[19/21]=0$ , 0 is stored in the variable  $\lambda_4$ ; since  
 $[<5/3>+<5/7>]=[2/3+5/7]=[29/21]=1$  and  $(-5)^{-1} \equiv 16^{-1} \equiv 4 \pmod{21}$ ,  $\sigma_4$  is stored in the variable  $\lambda_5$ ; since  
15  $[<8/3>+<8/7>]=[2/3+1/7]=[17/21]=0$ , 0 is stored in the variable  $\lambda_8$ ; since  $[<10/3>+<10/7>]=[1/3+3/7]=[16/21]=0$ , 0 is stored in the variable  $\lambda_{10}$ ; since  
 $[<11/3>+<11/7>]=[2/3+4/7]=[26/21]=1$  and  $(-11)^{-1} \equiv 10^{-1} \equiv 19 \pmod{21}$ ,  $\sigma_{19}$  is stored in the variable  $\lambda_{11}$ ; since  
20  $[<13/3>+<13/7>]=[1/3+6/7]=[25/21]=1$  and  $(-13)^{-1} \equiv 8^{-1} \equiv 8 \pmod{21}$ ,  $\sigma_8$  is stored in the variable  $\lambda_{13}$ ; since  
 $[<16/3>+<16/7>]=[1/3+2/7]=[13/21]=0$ , 0 is stored in the variable  $\lambda_{16}$ ; since  $[<17/3>+<17/7>]=[2/3+3/7]=[23/21]=1$  and  $(-17)^{-1} \equiv 4^{-1} \equiv 16 \pmod{21}$ ,  $\sigma_{16}$  is stored in the  
25 variable  $\lambda_{17}$ ; since  $[<19/3>+<19/7>]=[1/3+5/7]=[22/21]=1$  and  $(-19)^{-1} \equiv 2^{-1} \equiv 11 \pmod{21}$ ,  $\sigma_{11}$  is stored in the variable  $\lambda_{19}$ ; and since

$[<20/3>+<20/7>]=[2/3+6/7]=[32/21]=1$  and  $(-20)^{-1} \equiv 1^{-1} \equiv 1 \pmod{21}$ ,  $\sigma_1$  is stored in the variable  $\lambda_{20}$ , in the same way.

The total  $\omega = \sigma_4 + \sigma_{19} + \sigma_8 + \sigma_{16} + \sigma_{11} + \sigma_1$  of the whole data stored in the respective variables  $\lambda_1, \lambda_2, \lambda_4, \lambda_5, \lambda_8, \lambda_{10}, \lambda_{11}, \lambda_{13}, \lambda_{16}, \lambda_{17}, \lambda_{19}, \lambda_{20}$  is computed in Step S23 of Fig. 2. Here, the total means the total in the group ring  $Z[G(Q(\zeta)|Q)]$ , indicating the total of the coefficients of the respective  $\sigma_i$  with the respective  $\sigma_i$  regarded as symbols. The arithmetic result  $\omega$  is temporarily stored in the  $\omega$ -storing file 163 through the central processing unit 19.

The Jacobian addition candidate value computing device 12 obtains  $a=3, b=7, n=160, \omega=\sigma_4 + \sigma_{19} + \sigma_8 + \sigma_{16} + \sigma_{11} + \sigma_1$  from the a-storing file 161, the b-storing file 162, the n-storing file 168, and the  $\omega$ -storing file 163 and computes the candidate value  $j$  of the Jacobian addition, according to the processing as shown in Fig. 3, as follows.

Since  $ab=21$ , the primitive  $21^{\text{st}}$  power root of 1 is stored in the variable  $\zeta$ , and since  $2n / (a-1)(b-1) = 26.6\dots$ , 27 is stored in the variable  $m$ , in Step 31 of Fig. 3.

The random integer of the cyclotomic field  $Q(\zeta)$  is stored in the variable  $\gamma_0$  as follow, in Step S32 of Fig. 3. The variable  $\gamma_0$  is initialized to 0; the random number  $r_0 = -2$  is generated as for  $t=0, r_0 \zeta^0 = -2$  is



added to  $\gamma_0$ , so to require  $\gamma_0 = -2$ ; the random number  $r_1 = 2$  is generated as for  $t=1$ ,  $r_1 \zeta^1 = 2 \zeta$  is added to  $\gamma_0$ , so to require  $\gamma_0 = -2 + 2\zeta$ ; and the same operation is repeated until  $t=11$ , so to require  $\gamma_0 = -2 + 2\zeta - \zeta^2 + 2\zeta^3 + 2\zeta^5 - \zeta^6 - \zeta^7 - 2\zeta^8 + 2\zeta^9 - \zeta^{11}$ , in Step S32 of Fig. 3.

The following operation will be performed on the respective integers  $i=0, 1, 2, \dots$ , in Step S33 of Fig. 3. As for  $i=0$ ,  $\gamma_0 + 0$  is stored in the variable  $\gamma$ , so to obtain  $\gamma = -2 + 2\zeta - \zeta^2 + 2\zeta^3 + 2\zeta^5 - \zeta^6 - \zeta^7 - 2\zeta^8 + 2\zeta^9 - \zeta^{11}$ , to compute the norm  $N_{Q(\zeta)|Q}(\gamma)$ . The resultant 129571513 is stored in the p-storing file 167, the number of bits 29 of  $p = 129571513$  is stored in the variable 1. That  $1=29$  and  $m=27$  or so is confirmed. When  $p = 129571513$  is factorized into prime factors in the known way,  $p = 129571513 = 43 \times 211 \times 14281$  is obtained. Since  $p = 129571513$  is not a prime number, the operation as for  $i = 0$  is finished, and as for  $i = 1$ , the same operation will be repeated. In the form of this embodiment, the same operation is continued until  $i = 2$ . As for  $i = 2$ ,  $\gamma_0 + 2$  is stored in the variable  $\gamma$ , so to obtain  $\gamma = 2\zeta - \zeta^2 + 2\zeta^3 + 2\zeta^5 - \zeta^6 - \zeta^7 - 2\zeta^8 + 2\zeta^9 - \zeta^{11}$ , to compute the norm  $N_{Q(\zeta)|Q}(\gamma)$ . The resultant 163255597 is stored in the p-storing file 167, and the number of bits 28 of  $p = 163255597$  is stored in the variable 1. Since  $1=28$  and  $m=27$  or so, and  $p = 163255597$  is determined as a prime number (a known method is used

for judgment of a prime number), the operation of Step S33 will be finished.

In Step S34 of Fig. 3, the Stickelberger element  $\omega = \sigma_4 + \sigma_{19} + \sigma_8 + \sigma_{16} + \sigma_{11} + \sigma_1$  is adopted to work on the value  $2\zeta - \zeta^2 + 2\zeta^3 + 2\zeta^5 - \zeta^6 - \zeta^7 - 2\zeta^8 + 2\zeta^9 - \zeta^{11}$  of the variable  $\gamma$  and the result is stored in the j-storing file 164.

Namely, since  $j = \sigma_4(\gamma)\sigma_{19}(\gamma)\sigma_8(\gamma)\sigma_{16}(\gamma)\sigma_{11}(\gamma)\sigma_1(\gamma) = -11346 + 4158\zeta + 9337\zeta^2 - 10930\zeta^3 + 3060\zeta^4 + 11132\zeta^5 - 1408\zeta^6 - 10000\zeta^7 + 7506\zeta^8 + 1237\zeta^9 - 9894\zeta^{10} + 16406\zeta^{11}$ , the content of the j-storing file 164 becomes  $-11346 + 4158\zeta + 9337\zeta^2 - 10930\zeta^3 + 3060\zeta^4 + 11132\zeta^5 - 1408\zeta^6 - 10000\zeta^7 + 7506\zeta^8 + 1237\zeta^9 - 9894\zeta^{10} + 16406\zeta^{11}$ .

The order candidate value computing device 13 obtains a, b, j respectively from the a-storing file 161, the b-storing file 162, and the j-storing file 164, according to the processing as shown in Fig. 4, and computes each candidate value for the order of the Jacobian group, as follows.

Since  $ab=21$ , the primitive  $21^{\text{st}}$  power root of 1 is stored in the variable  $\zeta$ , in Step S41 of Fig. 4.

In Step S42 of Fig. 4,  $N_{Q(\zeta)|Q}(1 + (-\zeta)^k j)$  is computed as for the respective integers  $k=1, \dots, 2ab=42$ , by using the Jacobian addition candidate value j, and the result is stored in the variable  $h_k$ . Namely, since  $N_{Q(\zeta)|Q}(1 + (-\zeta)^k j)$

=18945750554224674862720917379214050968749547249577 as  
for  $k=1$ ,  
18945750554224674862720917379214050968749547249577 is  
stored in the variable  $h_1$ , and since  $N_{q(\zeta)}(1 + (-\zeta)^2 j)$   
5 = 18928969305265796978830941938772180777050417721949 as  
for  $k=2$ ,  
18928969305265796978830941938772180777050417721949 is  
stored in the variable  $h_2$ .

10                    Hereinafter, in the same way,  
18939442397757559639176586128404383479076142135761 is  
stored in the variable  $h_3$ ;  
18935060345406437247984249590121980321244862496761 is  
stored in the variable  $h_4$ ;  
15                    18935622676852726684902816970612470237474541809664 is  
stored in the variable  $h_5$ ;  
18931936903665705475581647305574444786263237069081 is  
stored in the variable  $h_6$ ;  
18929560654771860101383318185997674116929626012889 is  
20                    stored in the variable  $h_7$ ;  
18939150203650250186166315242126355786799280592469 is  
stored in the variable  $h_8$ ;  
18932675807273674693936115572103379669380378369473 is  
stored in the variable  $h_9$ ;  
25                    18942309965821405414970614992239749691042375170033 is  
stored in the variable  $h_{10}$ ;  
18934229290635176830764035532046510839791719442389 is

stored in the variable  $h_{11}$ ;

18935834172588603026508807514961653603431968293369 is

stored in the variable  $h_{12}$ ;

18938078743053945947831932134835899678969080710281 is

5 stored in the variable  $h_{13}$ ;

18930980854114698521197692341107826796840225368461 is

stored in the variable  $h_{14}$ ;

18925926348482126046797408190951930473609373791353 is

stored in the variable  $h_{15}$ ;

10 18936229724314338327608155999193464492913218459633 is

stored in the variable  $h_{16}$ ;

18935389098278487495205740285052812170943878823253 is

stored in the variable  $h_{17}$ ;

18931691567781542998050896522571358027374445665073 is

15 stored in the variable  $h_{18}$ ;

18932734180610926108166703609049207716180145717849 is

stored in the variable  $h_{19}$ ;

18938664411743724815803784593761801461579705647693 is

stored in the variable  $h_{20}$ ;

20 18933942752770105179837989473472080616474423254969 is

stored in the variable  $h_{21}$ ;

18919302986335777367049540268484273861903106390769 is

stored in the variable  $h_{22}$ ;

18936075396885270373781711765180522497408613713621 is

25 stored in the variable  $h_{23}$ ;

18925604328984592629627465194343191206594160037073 is

stored in the variable  $h_{24}$ ;

```
18929984863788418751836156261712299372083231633577 is
stored in the variable h25;
```

18929422531793648170111228339741198150094983499776 is  
stored in the variable  $h_{26}$ ;

5        18933107954541528865152848804062672753166448460761 is  
stored in the variable  $h_{27}$ ;

18935483634705487053043563594391048299333735703993 is  
stored in the variable  $h_{28}$ ;

```
18925896848340062851972136696783348221127455098349 is
stored in the variable h9,i;
```

18932368490475205159124453933007681555744686326777 is  
stored in the variable  $h_{30}i$

18922739336864448742750281538719599103232717642873 is  
stored in the variable  $h_{31}$ ;

15        18930815175217344826609492375186423724694014551957 is  
stored in the variable  $h_{32i}$

18929210510360406226057659372472230885175421077009 is  
stored in the variable  $h_{33}$ ;

18926967327936730178250537884862137815188718140673 is  
stored in the variable  $h_{34}$ ;

18934063763272126450623787600233843527396400812437 is  
stored in the variable  $h_{35}i$

18939120559761876801054292506881700885415287701041 is  
stored in the variable  $h_{36}i$

```
25      18928816315710623530089460607608797337081800632473 is
      stored in the variable h37i;
```

18929656538570982720438072809652072203857571941789 is

stored in the variable  $h_{38}$ ;  
18933352933862176606331230531189579186007983024249 is  
stored in the variable  $h_{39}$ ;  
18932310663274994445599743180032079937147687805121 is  
5 stored in the variable  $h_{40}$ ;  
18926381945702726406182624557022344113037957991709 is  
stored in the variable  $h_{41}$ ; and  
18931102681789095072229676262975577344314266433617 is  
stored in the variable  $h_{42}$ , respectively.

10  
Finally, the order candidate value computing  
device 13 combines the contents of the variables  $h_1$  to  $h_{42}$   
together as H and stores the same into the H-storing  
file 165.

15 The security judging device 14 obtains the H  
from the H-storing file 165, and searches for a  
candidate value h meeting the security condition of  
almost prime number characteristic from the order  
candidate values  $h_1, h_2, \dots, h_{42}$  included in the H, and  
20 stores the same into the h-storing file 166. In this  
form of the embodiment, for brief description, the  
security condition is considered as for the almost prime  
number characteristic. By use of the known prime number  
judging method,  $h_{11} =$   
25 18934229290635176830764035532046510839791719442389 is  
judged to be a prime number, and the security judging  
device 14 stores  $h = h_{11} =$

18934229290635176830764035532046510839791719442389 into the h-storing file 166.

5 The parameter deciding device 15 obtains a, b, p, and h respectively from the a-storing file 161, the b-storing file 162, the p-storing file 167, and the h-storing file 166, and operates according to the processing as shown in Fig. 5.

10 In Step S51 of Fig. 5, 127994587 that is the primitive 3<sup>rd</sup> power root of 1 with p = 163255597 used as the divisor is stored in the variable  $\zeta_3$ , and 8342648 that is the primitive 7<sup>th</sup> power root of 1 with p = 163255597 used as the divisor is stored in the variable  $\zeta_7$ .

15 In Step S52 of Fig. 5, the following processing will be performed on the respective integers l=1, 2, 3 and the respective integers m=1, 2, 3, 4, 5, 6, 7.

20 When l=1 and m=1,  $\zeta_3 = 127994587$  is stored in the variable  $\varepsilon$ , and  $\zeta_7 = 8342648$  is stored in the variable  $\eta$ . The random element  $\{151707017 + 104678491 x + 123646083 x^2 + 18753988 y + 87634493 x^3 + 61274336 x y + x^4, 138799785 + 145105684 x + 584395 x^2 + 80828873 y + 34715892 x^3 + 121885874 x y + 59787844 x^4 + x^2 y,$   
25  $161162224 + 117150097 x + 100956100 x^2 + 89380061 y + 140032555 x^3 + 43367019 x y + y^2 \}$  of the Jacobian group of an algebraic curve defined by the expression  $\varepsilon y^3 + \eta$

$x^7 + 1 = 127994587 y^3 + 8342648 x^7 + 1 = 0$  is generated,  
and this is stored in the variable G, the power of  
 $h=18934229290635176830764035532046510839791719442389$  in  
the Jacobian group, on a point stored in the variable G  
5 is computed, and the result  $\{133659497 + 103424746 x +$   
 $136032897 x^2 + 131029199 y + 24618867 x^3 + 114944034 x y$   
 $+ x^4, 86125426 + 125891893 x + 19568269 x^2 + 27044314 y +$   
 $80420960 x^3 + 137562092 x y + x^2 y, 53604112 + 65990501 x$   
 $+ 51269221 x^2 + 55271502 y + 7974233 x^3 + 84922220 x y +$   
10  $y^2 \}$  is stored in the variable G.

Since the above content of the variable G is not  
equal to the identity element  $\{ \}$  in the Jacobian group,  $\zeta$   
 $\zeta_3 = 127994587$  is stored in the variable  $\varepsilon$ , and  $\zeta_7^2 =$   
 $8342648^2 \bmod 163255597 = 159772073$  is stored in the  
15 variable  $\eta$ , as for  $l = 1$  and  $m = 2$ , thereby repeating  
the above processing.

In the case of the form of this embodiment, when  
 $l = 2$  and  $m = 2$ ,  $\varepsilon = 35261009$  and  $\eta = 159772073$  are  
obtained, the power of

20  $h=18934229290635176830764035532046510839791719442389$  on  
the point  $G = \{4568071 + 141843715 x + 68256743 x^2 +$   
 $71903501 y + 128953783 x^3 + 10781960 x y + x^4, 48272788 +$   
 $45615229 x + 150692034 x^2 + 53973350 y + 11114765 x^3 +$   
 $78550130 x y + 61331354 x^4 + x^2 y, 117552807 + 135448907$   
25  $x + 64074711 x^2 + 141058974 y + 49208246 x^3 + 93940317 x$   
 $y + y^2 \}$  generated at random results in the identity  
element  $\{ \}$ , and the parameter deciding device 15



supplies  $p=163255597$ ,  $\varepsilon=35261009$ ,  $\eta=159772073$  as the parameters of a secure algebraic curve.

Finally, the parameters  $p=163255597$ ,  $\varepsilon=35261009$ ,  $\eta=159772073$  supplied by the parameter deciding device 15 are supplied from the output device 18.

This time, the form of a second embodiment according to the present invention will be described in detail.

The form of the second embodiment according to the present invention is a secure parameter generating method in an algebraic curve cryptography, comprising:

- (a) a Stickelberger element computing procedure of requiring the prime numbers  $a$  and  $b$  respectively from the  $a$ -storing file 161 and the  $b$ -storing file 162 and computing the Stickelberger element  $\omega$  in the  $ab$  portion of the cyclotomic field;
- (b) a procedure of storing the Stickelberger element  $\omega$  computed in the above Stickelberger element computing procedure into the  $\omega$ -storing file 163;
- (c) a Jacobian addition candidate value computing procedure of requiring the prime number  $a$ , the prime number  $b$ , the size  $n$  of an encryption key, and the Stickelberger element  $\omega$  respectively from the  $a$ -storing file 161, the  $b$ -storing file 162, the  $n$ -storing file 168, and  $\omega$ -storing file 163 and computing the Jacobian addition candidate value  $j$  as for the two different prime numbers  $a$  and  $b$  and the prime number  $p$

corresponding to the Jacobian addition candidate value  
j;

(d) a procedure of storing the prime number p and the  
Jacobian addition candidate value j computed in the  
5 above Jacobian addition candidate value computing  
procedure, respectively into the p-storing file 167 and  
the j-storing file 164;

(e) an order candidate value computing procedure of  
requiring the prime number a, the prime number b, and  
10 the Jacobian candidate value j respectively from the a-  
storing file 161, the b-storing file 162, and the j-  
storing file 164 and computing the class H consisting of  
a plurality of candidate values for the order of the  
Jacobian group of the algebraic curve specified by the  
prime number a and the prime number b;

(f) a procedure of storing the class H computed in the  
above order candidate value computing procedure into the  
H-storing file 165;

(g) a security judging procedure of requiring the class  
20 H from the H-storing file 165 and searching for the  
candidate value h meeting the security condition such as  
almost prime number characteristic;

(h) a procedure of storing the candidate value h found  
in the above security judging procedure, into the h-  
25 storing file 166; and

(i) a parameter deciding procedure of requiring the  
prime number a, the prime number b, the prime number p,

and the candidate value  $h$  respectively from the a-storing file 161, the b-storing file 162, the p-storing file 167, and the h-storing file 166 and computing a parameter of an algebraic curve whose order of the Jacobian group is in accord with the candidate value  $h$ , of the algebraic curves specified by the prime number  $a$ , the prime number  $b$ , and the prime number  $p$ .

The form of a third embodiment according to the present invention will be described in detail with reference to the drawings, this time. Fig. 6 is a block diagram showing the form of the third embodiment according to the present invention.

With reference to Fig. 6, the form of the third embodiment of the present invention is a storing medium 130 for storing a program for running the respective procedures according to the form of the second embodiment of the present invention, on a computer 100. This program is executed after being loaded in a storage of the computer 100.

The present invention is effective in enabling the use of a higher and complicated algebraic curve which couldn't be used, for an algebraic curve cryptography, and improving the security of the algebraic curve cryptography. This is because an algebraic curve having a prime number substantially as the order of the Jacobian group can be found efficiently from the class of algebraic curves having the following

definition equation;  $\alpha Y^a + \beta X^b + 1 = 0$ , thereby expanding the range of the usable algebraic curves and dispersedly increasing the decoding work of a hacker.

Although the invention has been illustrated and described with respect to exemplary embodiment thereof, it should be understood by those skilled in the art that the foregoing and various other changes, omissions and additions may be made therein and thereto, without departing from the spirit and scope of the present invention. Therefore, the present invention should not be understood as limited to the specific embodiment set out above but to include all possible embodiments which can be embodied within a scope encompassed and equivalents thereof with respect to the feature set out in the appended claims.